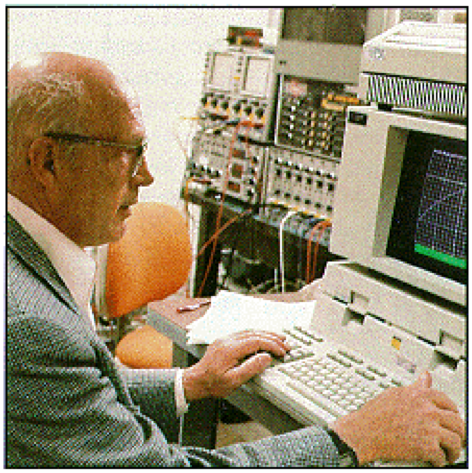
Bently's Corner

More about that damping term



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The accepted standard basic equation for response as a result of imbalance of a simple rotor system is

$$(1)\ d = \frac{(2000)(W_u)(r_u)(\omega^2)\ \angle A_u - A}{g\sqrt{(K-\omega^2M)^2 + (\omega D)^2}}$$

$$(2)\ A = \arctan\left(\frac{\omega D}{K-\omega^2 M}\right)$$

where

- d is rotor 1X (synchronous) lateral displacement amplitude in pp mils
- W_u is weight of residual imbalance in pounds
- r_u is radius of imbalance in inches
- ω is rotative speed in rad/sec.
- g is gravity acceleration, 386 in./sec.^{2}
- K is effective (first bending mode) rotor system stiffness coefficient in lbs./in.
- M is effective (first mode) rotor mass in lb. sec.^{2}/in.
- D is fluid lubricated bearing viscous damping coefficient in lb. sec./in.
- A_u is the angular location of the residual imbalance
- A is the angular relationship of the high spot to the heavy spot
- $\angle A_u - A$ indicates the angular position of the response vector.

This is a very good equation for a monkey on a bungee. But, unfortunately, it is not quite complete for the synchronous response of even a very simple rotor system.

What is required to correct equations (1) and (2) is the inclusion of the effect at synchronous speed of the bearing/seal fluid average velocity ratio λ, which adds to dynamic stiffness the term corresponding to the famous "cross coupled spring," which in turn drives oil whirl and other aerodynamic instabilities. Evaluated at synchronous speed, this term is directly proportional to damping and equals $-\lambda\omega D$ where minus indicates its direction opposite to damping force and λ is fluid average velocity ratio.

The correct basic synchronous response is then

$$(3)\ d \approx \frac{(2000)(W_u)(r_u)(\omega^2)\ \angle A_u - A}{g\sqrt{(K-\omega^2M)^2 + (\omega D - \lambda\omega D)^2}}$$

$$(4)\ A = \arctan\frac{\omega D(1-\lambda)}{K-\omega^2 M}$$

Since λ is generally slightly less than 1/2 (though in specific situations, such as on some pumps, it has been observed as high as 0.86), the basic equation for imbalance of a rotor system is

$$(5)\ d \approx \frac{(2000)(W_u)(r_u)(\omega^2)\ \angle A_u - A}{g\sqrt{(K-\omega^2M)^2 + .25(\omega D)^2}}$$

$$A \approx \arctan\frac{(.5\omega D)}{(K-\omega^2 M)}$$

and yields the Synchronous Amplification Factor (Q_{sy}) as follows:

$$(6)\ Q_{sy} = \frac{M\omega_{CR}}{D(1-\lambda)} \approx \frac{2M\omega_{CR}}{D}$$

(Previously, this has been

$$Q_{sy} = \frac{M\omega_{CR}}{D} \text{ where } \omega_{CR} = \sqrt{K/M} \text{ is}$$

the rotor natural frequency corresponding to the first bending mode.)

There are two conclusions to be noted. The first is that, unless λ is known to be zero (it occurs at shaft high eccentricity ratios), the actual Synchronous Amplification Factor Q_{sy} can be doubled by the effect of "cross coupled spring." The second is essentially a philosophical point of basic understanding of typical rotor behavior explained below.

When the rotor performs nonsychronous precession, equation (3) shows a denominator of

$$(7)\ \sqrt{(K-\omega_p^2M)^2 + (\omega_p D - \lambda\omega D)^2}$$

where ω_p is the rotor angular precessional speed. Everyone knows that the balance resonance speed ("first critical") occurs when the $(K-\omega^2M)$ terms go to zero, which yields $\omega = \omega_p = \omega_{CR} = \sqrt{K/M}$. What is not generally known is that the whirl resonance occurs when the other component of (7), namely $(\omega_p D - \lambda\omega D)$ becomes zero, i.e. it yields $\omega_p = \lambda\omega$, which represents another "critical" speed (rotor/bearing system natural frequency). This simple observation alleviates some of the unnecessary deep mysteries of rotor system instabilities.

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Editor's Note: Mr. Bently also wrote on damping in the July 1984 *Orbit*. For a copy of the article, "What happened to the damping?," please request L8117. ■